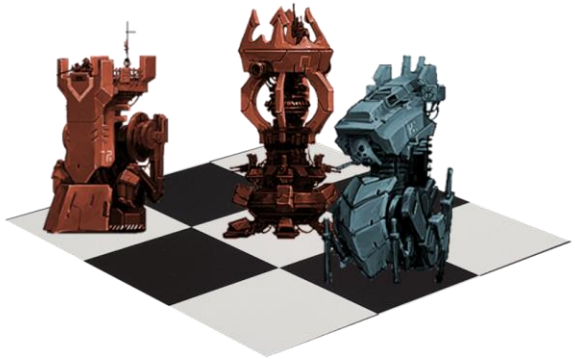
Introduction to Mathematical Game Theory



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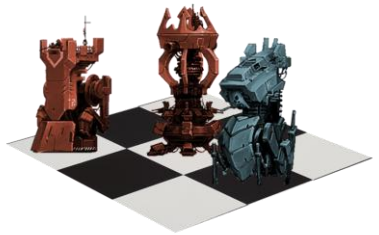

Introduction to Mathematical Game Theory

Mathematical Game Theory deals with mathematical techniques able to analyze situations in which two or more individuals make decisions to influence their and other's benefits.

Situations studied by Game Theorists are not only recreational as the "*game*" word should suggest, often Game Theory is applied to Industrial, Business, Geo Political and Military Problems

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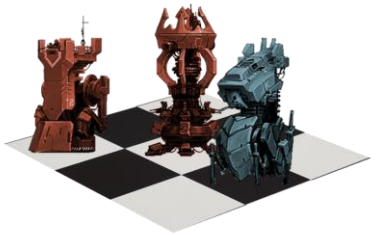
Origins of Game Theory



We date the beginning of this modern Theory with the paper of Von Neumann & Morgenstern in 1944 (*Games and Economic Behavior*), the word game is applied to any social situation involving two or more individuals: *the Players*.

The Players are rational Decision Makers:

they take decisions to maximize the payoffs of their expected utility

Game Theory develops methods and techniques to play Games and to identify Winning Strategies

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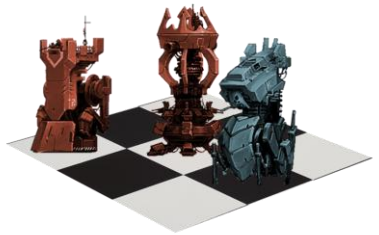
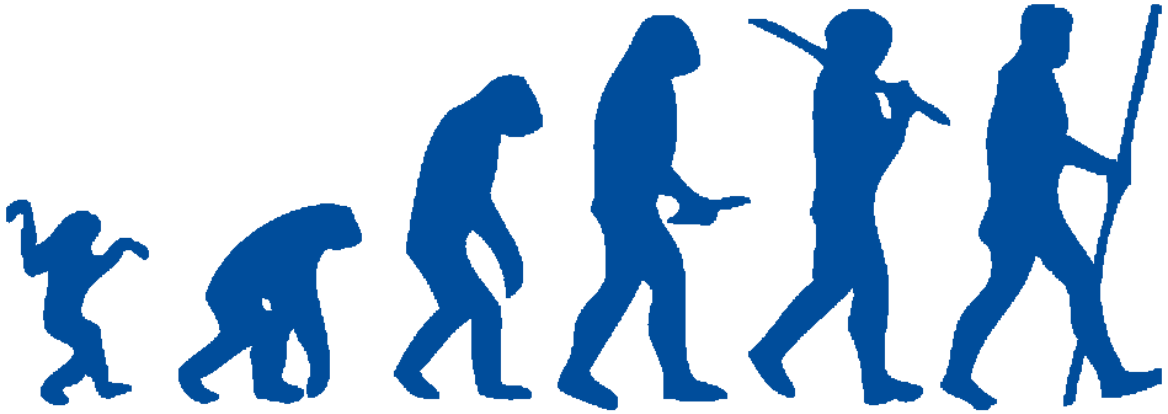
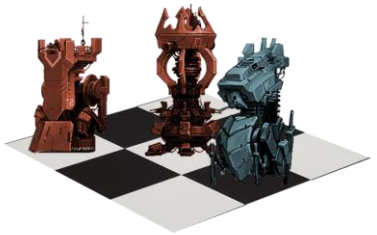
Payoff & Evolution



An example of behavior maximizing its payoff could be easily found in the Evolution Models, inspired by Darwin's Theory.
In an Universe where the increasing disorder is a Physics Law, the complex organisms (including humans or in general the social organizations) can survive only if they have a Behavior to *enhance their probabilities of surviving and reproducing*



Payoff as Complex Function



Currently. the Evolutionistic Selection Argument suggests that individuals want to maximize the expected value of a Measure related to natural surviving and reproductive suitability, otherwise they disappear (see Maynard Smith 1982).

In general maximizing the expected Utility Payoff is not just the same that maximizing expected Money Payoff, because the Values of Utility are not always expressed just in Dollars or Euros

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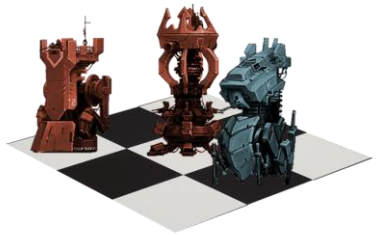
Decision Maker Attitude



A Risk Adverse Player enhances its expected utility more by winning a dollar when he/she is poor then when he/she is rich

It's never too early to start winning
It is always too early to start losing
J.Fleming, 007 Goldfinger

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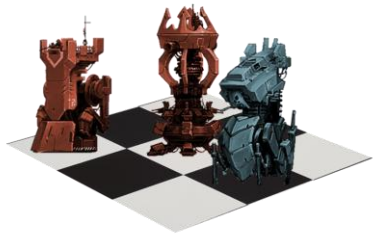

Decision Theory

Winston Churchill
UK PM during WWII

Game Theory can be seen as a generalization of **Decision Theory** (with two or more decision makers), so to understand well the fundamental concepts of **Game Theory** it is necessary to study the basics of **Decision Theory**

Doug McMillion
Wall Mart CEO 2020

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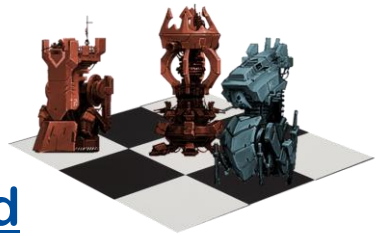

Identify Goals & Objectives

Decisions fill up our life and the Capacity of Choosing and Expressing our whishes are the points which make the difference between the Life of an Intelligent Being from an Inferior Forms.

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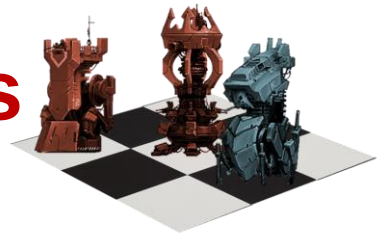
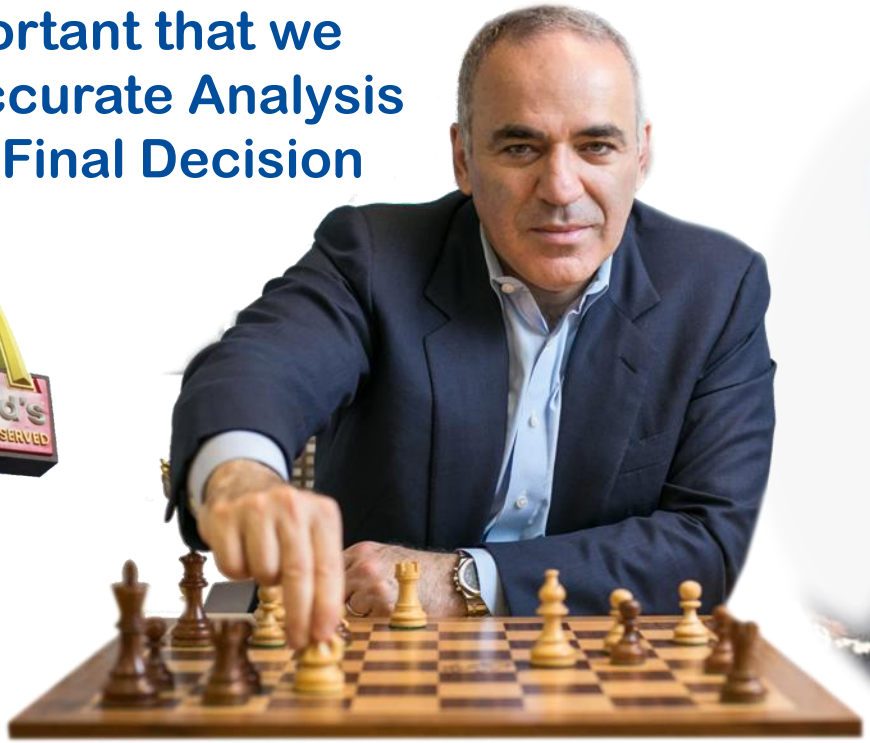


Different Decisions & Games

Every Day we make Decisions, but some of them have so small importance that we forget them within a few minutes, while others are so important that we conduct a Very Accurate Analysis before to take the Final Decision



Garry K.Kasparov, Legendary World Chess Champion

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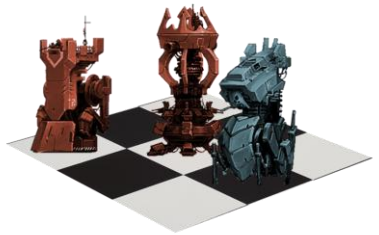
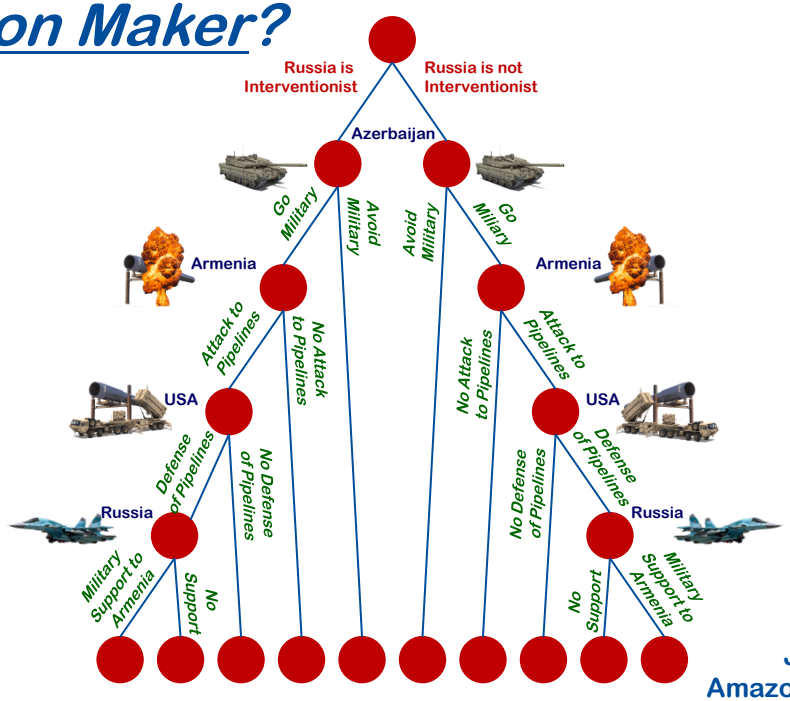


Analysis and People in Game Theory

What is an accurate Analysis?

Who is a good *Decision Maker?*

	Campaign	No Campaign
Campaign	2.5;2.5	4.0;2.0
No Campaign	2.0;4.0	3.3;3.3



Jeff Bezos
Amazon Founder

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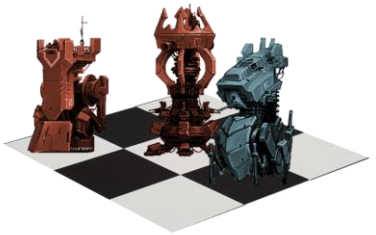
Game Theory Matrix Example

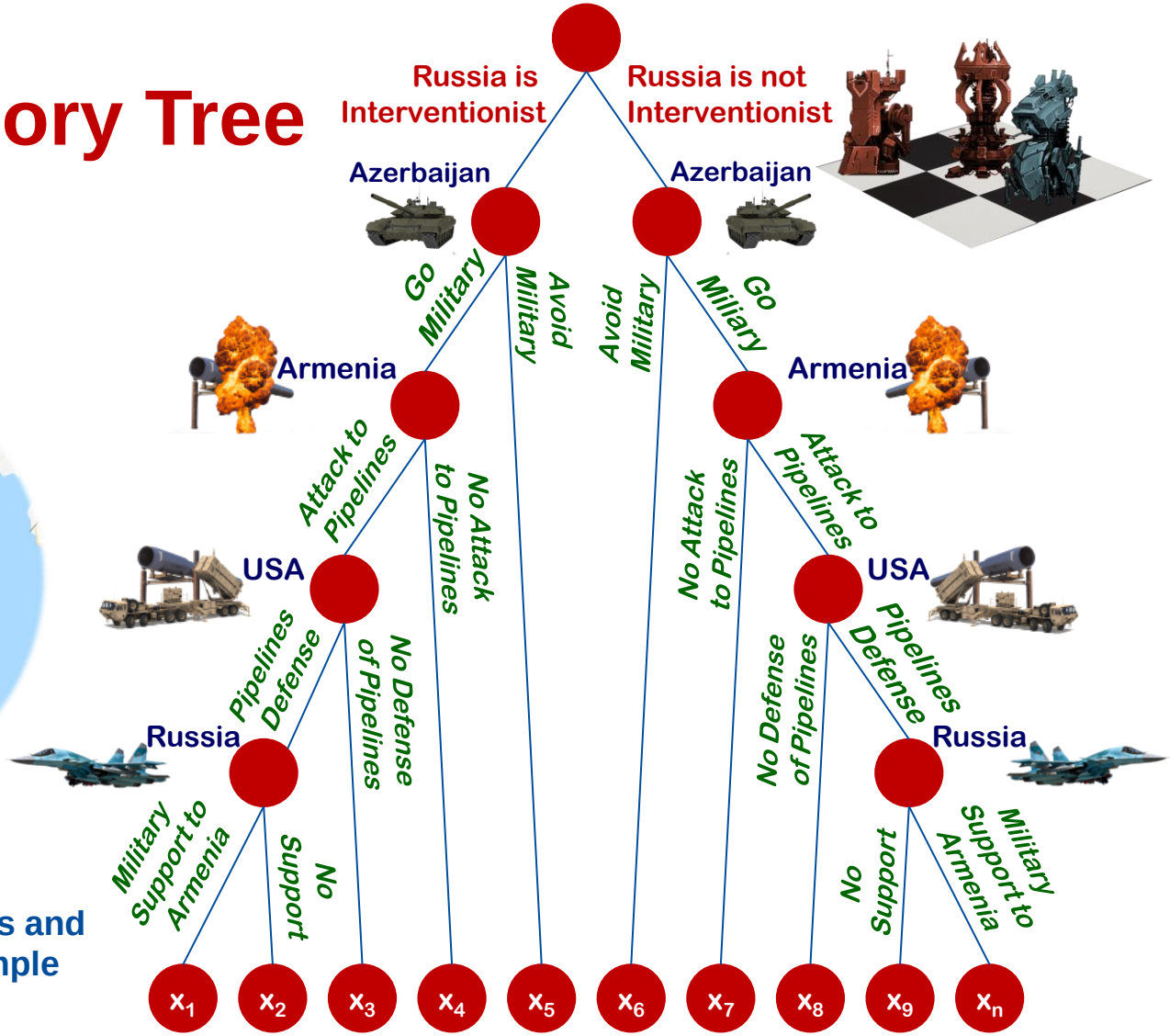


	Campaign	No Campaign
Campaign	2.5;2.5	4.0;2.0
No Campaign	2.0;4.0	3.3;3.3

Payoff expressed as Revenues in Canada in bUSD based on Commercial Promotions

This is just a Fictional Scenario without any reference to real events and created ad hoc to propose an Example

Game Theory Tree Example



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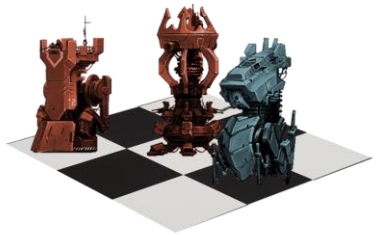
Delegate or Decide?



Sometimes we have precise ideas about who is a Good Decision Maker, then we will be able to criticize the decisions of our delegates.

Often we have to delegate others and we would like to be sure they decide well, but only if we are able to recognize a good Decision Maker, we can be really sure of this.

Deep Blue by IBM
1st Computer able to win
against the human Chess World Champion

One goal of Mathematical Game Theory is to build Mathematical Tools to make Good Decisions and interact Strategically with others' Decisions

Luigi Einaudi
President of Italy
1948-1955

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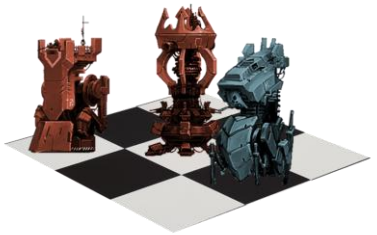

Cooperative & Not Cooperative Games

Two very important frameworks in Game Theory are:

- **Non Cooperative Games**
- **Cooperative Games.**

These two interesting kinds of Games are very different respect the Mathematical Tools that we need to use in order to properly address and solve them

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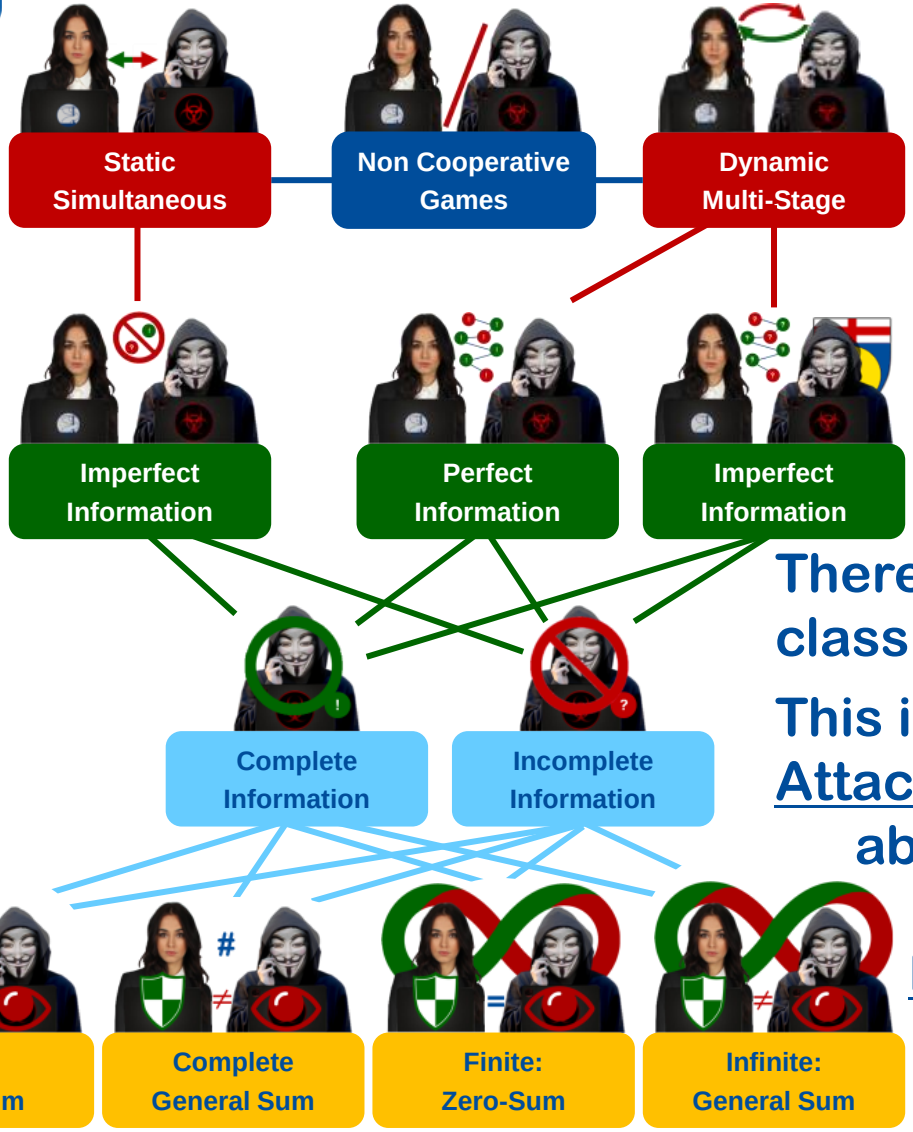

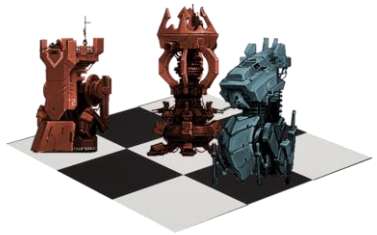

Classification of Games

There are many different ways to classify Games

This is an example related to Cyber Attacks and include considerations about Static & Dynamic Games, Games with Perfect & Imperfect Info, Complete & Incomplete Games, Zero Sum & General Sum Games

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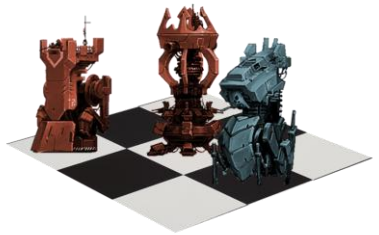

"*Chess*" (Persian شاه shāh, *King*) is a Strategy Game born around VI in India and turned popular in Europe around 1000 AD.

It has taken up to February 10, 1996 to get a World Champion (i.e. G.K. Kasparov) to be defeated by a Computer (Deep Blue from IBM) able to look forward up to 16 billion moves

La partita a scacchi di Gerolamo Induno. Galleria d'arte moderna, Milano. Artista: Brogi (1915-1920 ca.)

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